6

Question 1: (10 points)

The downtime per day for a certain computing facility averages 250 minutes with a standard deviation of 50 minutes. The daily downtime was observed for a randomly selected period of 36 days. Find the probability that the average daily downtime is between 240 and 300 minutes:

$\bar{X} = 250$ min

$S = 50$ min.

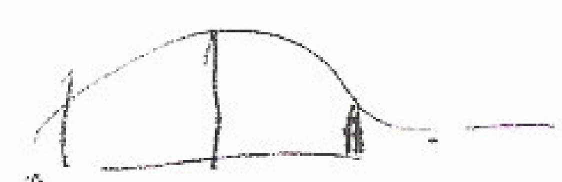
$n = 36$

$$z = \frac{\bar{x} - \bar{X}}{S/\sqrt{n}}$$

why?

$$P(240 < \bar{X} < 300) = \left(\frac{240-250}{50/\sqrt{36}} < z < \frac{300-250}{50/\sqrt{36}}\right)$$

$$= (-1.2 < z < 6)$$



$A_{1.2} = 0.1151$

$$= (\overset{A_{1.2}}{0.3849} < z < \overset{A_6}{0})$$

$$= 0.5 - 0.3849 = \boxed{0.1151}$$

23

Question 2: (25 points)

A medical research worker wants to estimate the mean blood pressure of women in their fifties. Based on a previous study, he knows that $\sigma = 10.5$mm of mercury. The research worker takes a random sample of 120 women and finds $\bar{x} = 141.8$ mm of mercury.

a) What is the point estimate value of the true average blood pressure of women in their fifties? (4 points)

4

$\sigma = 10.5$ $n = 120$ $\bar{x} = 14.18$

The point estimate value $= \bar{X} = 14.18$

b) Construct a 98% confidence interval for the mean blood pressure of women in their fifties. (12 points)

Since n is large and we assume the pop. is normally distributed, then the interval will belong to

$$\left[\bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}},\ \bar{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]$$

$z_{\alpha/2}$: 98% confident $\rightarrow 1-\alpha = 0.98$ $\alpha = 0.02$

$z_{\alpha/2} = z_{0.01} = 2.33$

10

$$\left[14.18 - 2.33\left(\frac{10.5}{\sqrt{120}}\right),\ 14.18 + 2.33\left(\frac{10.5}{\sqrt{120}}\right)\right]$$

139.56 , 144.04

$[11.95,\ 16.41]$

we are 98% confident that the mean blood pressure of women in their fifties is between 11.95 and 16.41.

c) What is the maximum error of estimate for the resulting confidence interval? (3 points)

maximum error of estimate $= \dfrac{16.41 - 11.95}{2} = 2.23$

3!

d) How large the sample should be, if we want the maximum error of estimate to be maximum 1 mm of mercury? (6 points)

$$n \geq z_{\alpha/2}^2 \frac{\sigma^2}{\epsilon^2}$$

$$n \geq (2.33)^2 \frac{(10.5)^2}{1^2} \geq 598.53$$

6

$$n \geq 599$$

The sample should be $n \geq 599$.

13

Question 3: (15 points)

In producing resistors, variability of resistance is important, as it reflects the stability of the manufacturing process. Construct a 90% confidence interval for "σ", the true standard deviation of resistance measurements, knowing that a random sample of 15 resistors showed resistance with a standard deviation of 0.5 ohm.

$n = 15$. 90% conf. int why χ^2?

$S = 0.5$

conf. int. for $\sigma \in \left[\sqrt{\dfrac{(n-1)s^2}{\chi^2_{\alpha/2}}}, \sqrt{\dfrac{(n-1)s^2}{\chi^2_{1-\alpha/2}}}\right]$

$\cancel{(15-1)(0.5)^2}$ $\chi^2_{\alpha/2}$: $1-\alpha = 0.90 \Rightarrow \alpha = 0.1$

$\chi^2_{\alpha/2}(df-1) = \chi^2_{0.1/2}(14) = 23.685$

$\chi^2_{1-\alpha/2}$: $\chi^2_{1-0.1/2}(14) = 6.571$

$$\left[\sqrt{\frac{(15-1)(0.5)^2}{23.685}}, \sqrt{\frac{(15-1)(0.5)^2}{6.571}}\right] = [0.384, 0.73]$$

we are 90% confident that ~~a random sample of 15 resistors~~ the interval is between 0.384 and 0.73.

Question 4: (20 points)

The security department of a factory claims that the average time required by the night guard to walk his round is greater than 30 minutes. In a randomly selected sample of 16 rounds, the night guard averaged 30.8 minutes, with a standard deviation of 1.5 minutes.

20

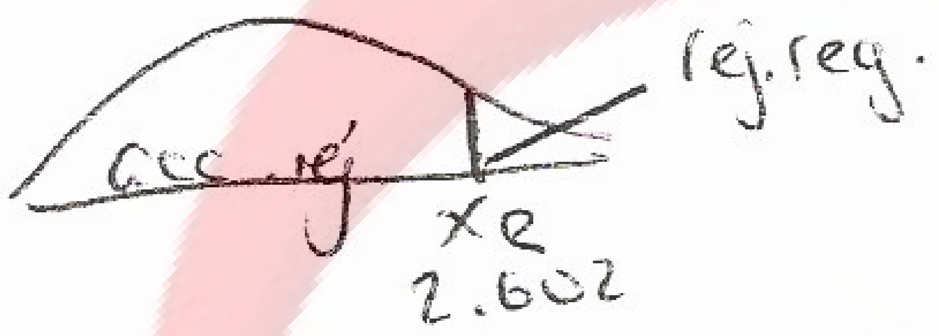
a) At $\alpha = 0.01$, is there sufficient evidence to reject the claim? (14 points)

$H_0: \mu = 30$ $\quad\quad$ $H_a: \mu > 30$

$n = 16 \quad \bar{X} = 30.8 \quad S = 1.5$

Since n is small ($n \leq 30$), ~~then~~ and we assume that the pop. is normal but σ is not given, then $\frac{\bar{X} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = t$ dist.

This is a one sided test with $X_R = t_\alpha^{(df = n-1)} = t_{0.01}^{(15)} = 2.602$



$OVTS = \frac{\bar{X} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{30.8 - 30}{1.5/\sqrt{16}} = 2.133$

14

OVTS is in the acceptance region, so we accept Ho and we reject the claim.

So there is sufficient evidence to reject the claim

b) Find the p-value for the above test. Based on this p-value would you reject the claim at $\alpha = 0.05$? Explain. (6 points)

Since it is a one sided test then.

6

$Pvalue = P(t_\alpha^{(15)} > |OVTS|)$

$= P(t > 2.133)$

$= 0.025$

$Pvalue < \alpha$

$0.025 < 0.05$ so we reject Ho.

and acc. Ha so we don't have enough evidence to reject the claim.

7.5

Question 5: (15 points)

A manufacturer of resistors claims that at most 10% of the produced resistors fail to meet the required specifications. In a random sample of 125 of these resistors, 17 fail to meet the specifications. At $\alpha = 0.1$, is there sufficient evidence to reject the manufacturer's claim? Use the p-value method.

$n = 125$. $P \leq 0.1$ $\alpha = 0.1$ $q = 1 - 0.1 = 0.9$

$\hat{P} = 17/125 = 0.136$

$H_0: P \geq 0.1$ $H_a:$ $P \neq 0.1$ $P > 0.1$

$P = 0.1$

since n is large and we assume ~~the population~~ is normal.

then $\frac{\hat{P} - \mu_{\hat{P}}}{\sigma_{\hat{P}}} = \frac{\hat{P} - P}{\sqrt{\frac{pq}{n}}} = z$ $\frac{0.136 - 0.1}{\sqrt{\frac{0.1(0.9)}{125}}} = \frac{0.036}{0.027} = 1.33$

Pvalue method: since it is a 2 sided test then

Pvalue $= 2P(z > OVTS) = 2P(z > 1.33)$

$= 2(0.5 - 0.4082) = 0.1836$

Since Pvalue $> \alpha$ $0.1836 > 0.1$ then we accept H_0 and $\cancel{ac}$ the claim. so there isn't enough evidence to rej. it

13

Question 6: (15 points)

In a large company, a researcher claims that the variances of the salaries of male employees and female employees are equal. To check this, a survey of salaries of 16 males and 21 females showed following:

Males:	$\bar{x} = 43475$	S = 400
Females	$\bar{x} = 39980$	S= 700

Assuming that the two populations from which we selected our samples are normally distributed, is there sufficient evidence to accept the claim? Test at α= 0.05.

$\boxed{H_0: \sigma_1^2 = \sigma_2^2}$ $H_a: \sigma_1^2 \neq \sigma_2^2$

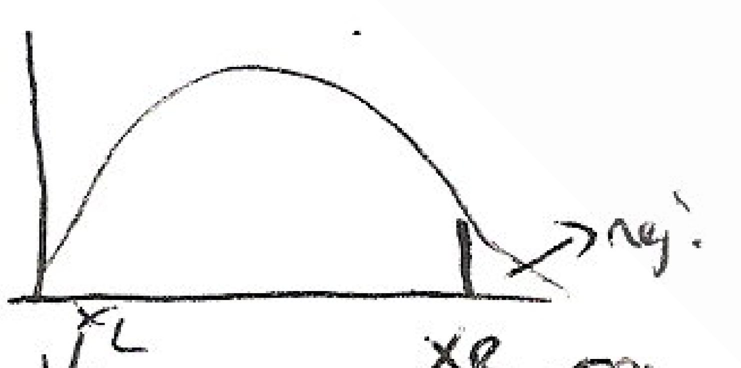
$n_2 = 16$ $n_1 = 21$

$\bar{X}_2 = 43,475$ $\bar{X}_1 = 39980$

$S_2 = 400$ $S_1 = 700$

$S_2^2 = 160,000$ $S_1^2 = 490,000$.

since the pop. are normal then $\frac{S_1^2}{S_2^2} = f = \frac{490,000}{160,000} = 3.0625$



$X_R: f_{\alpha/2}(df_1, df_2) = f_{0.025}(20, 15) = 2.7559$

we don't need to calculate it since $\frac{S_1^2}{S_2^2} > 1$

since $\frac{S_1^2}{S_2^2}$ is in the rejection region, so we reject H_0. and $\cancel{ac}$ reject the claim. because there isn't suff. evidence to accept the claim.